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An econometric approach of risk model in insurance

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Abstract: The paper presents a process for determining the minimum risk margin under a simple risk assurance model for goods insurance, which also takes account of insurers' risk of becoming insolvent.

Keywords: insurance, risk reserves, probability production, probability of ruining, probabilistic model, binomial model, transition function of probability, economic and social conjuncture

JEL Codes: C10, C13

Introduction

The authors consider the following simple issue after regarding the insurance of goods [5].

Problem. It is ensured $n \in \mathbb{N}^*$ goods of the same kind, the sum insured for each being S u.m. The probability of sinister's production is assumed to be $p \in \mathbb{Q}_+^*$. We determine the minimal risk threshold for the probability of ruining α.

It is known that solving such a problem implies consideration of the probability model of the sum insured for one good i, $1 \leq i \leq n$, as being the random variable X_i with the distribution:

$X_i: \begin{pmatrix} 0 & S \\ q & p \end{pmatrix}$, $q = 1-p$, $p \in \mathbb{Q}_+^*$.

In fact, the sum insured for the good *i* is the insured monetary compensation for the good *i*, in the case of a sinister one, which the insurance company has to pay to the insured.

Further the following hypotheses are used.

I1. X_1,...,X_n are independent random variables with the same distribution;
I2. The average value principle is applied, according to which the premium of insurance is equal to the average compensation.

Based on the hypothesis I1, the total compensation paid by the insurer in the case of sinister is modeled by the random variable $Y=\sum_{i=1}^{n} X_i$, which has:
$M(Y) = n \cdot p \cdot S,\ V(Y) = n \cdot p \cdot q \cdot S^2$ şi $\sigma=S\cdot\sqrt{npq}$.

Based on the I2 hypothesis, the total premium of insurance , denoted by Pnt is:
Pnt= $M(Y) = n \cdot p \cdot S$.

Then **the reserve fund**, denoted by R, is defined by inequality:
$P(Y - \text{Pnt} > R) \leq \alpha,$
where α is an accepted value, for example $\alpha \in \{0{,}005; 0{,}001; 0{,}01\}$.

The algorithm for determining the minimal threshold of the risk reserve, as demonstrated in [5, p.115], is:
Step 1. Determine from the table of the normal distribution $z_{1-\alpha}$, the quantile of order 1-α;
Step 2. Determine
$\sigma=\sqrt{V(Y)}$;
Step 3. Determine the minimal threshold of the risk reserve, denoted by $R_{\min}$, with the formula:
$R_{\min} = \sigma \cdot z_{1-\alpha} \cdot \sqrt{n}$.

The econometric approach of the probabilistic modeling

a) First, it is found that solving the above problem uses the random variables X_1,...,X_n, without indicating the probability field where they are defined, so the probability P from the definition of the reserve fund is not specified.
b) Secondly, the probabilistic modeling used does not take into account the risk that in the near future, some insured become insolvent, as the socio-economic environment has changed.

To eliminate the objection a) observing that any random variable X_i has two values (is dichotomous), we will attach his, in a canonical way, a urn U with the balls of two color, which has the probabilistic model the binomial model (B,$P(B)$, **P**), where $B=\{\omega_1, \omega_2\}$, $P(B)$ is the set of subsets of the B, **P**: $P(B)\rightarrow[0,1]$, **P**($\{\omega_1\}$)=q, **P**($\{\omega_2\}$)=p, X_i: (B,$P(B)$, **P**)$\rightarrow(\mathbb{R}, \mathcal{B}_{\mathbb{R}})$, $X_i(\omega_1) = 0$, $X_i(\omega_2) = S$, **P**($X_i = 0$)=**P**($\{\omega_1\}$)=q şi **P**($X_i = S$)= **P**($\{\omega_2\}$) =p.

To eliminate the objection b) we believe that, in general, the clients of an insurance company live in a time-varying socio-economic environment, he can cross various conjunctures, among which for some may be favorable, in the sense that they allow them to pay the premiums of insurance and in this case, such a conjuncture will be called **favorable** and marked by CF.

Other circumstances may be **unfavorable**, such unfavorable circumstance being marked by CNF.

We further consider the following dichotomous hypothesis.

I3. A conjuncture is either favorable or unfavorable.
According to the hypothesis I3, if Z is the random variable that probabilistically models the state of the socio-economic environment in which there are clients, then his repartition is:
Z:$\begin{pmatrix} 0 & 1 \\ \beta & \gamma \end{pmatrix}$, β=1-γ, $\gamma\in\mathbb{Q}_+^*$,
where 0 denotes CNF, and 1 denotes CF.
We also use the following hypothesis.

I4. $n\cdot p = m \in\mathbb{N}^*$ şi $n\cdot \gamma = \delta \in\mathbb{N}^*$, n being the number of insured goods. Based on the I4 hypothesis we assign to the random variable Z a Bernoulli urn U_Z with n-δ red balls and δ yellow balls, which has the probabilistic model the binomial model $(B_Z, P(B_Z), \mathbf{P_Z})$, where $B_Z=\{\omega_1,\omega_2\}$, $P(B_Z)$ is the set of subsets of the B_Z, $\mathbf{P_Z}$: $P(B_Z)\rightarrow[0,1]$, $\mathbf{P_Z}$ $(\{\omega_1\})=\beta$, $\mathbf{P_Z}$ $(\{\omega_2\})=\delta$, Z: $(B_Z, P(B_Z), \mathbf{P_Z})\rightarrow(\mathbb{R}, \mathcal{B}_{\mathbb{R}})$, $Z(\omega_1)=0$, $Z(\omega_2)=1$, $\mathbf{P_Z}$ $(Z=0)$= $\mathbf{P_Z}$ $(\{\omega_1\})=\beta$ and $\mathbf{P_Z}$ $(Z=1)$= $\mathbf{P_Z}$ $(\{\omega_2\})=\gamma$.

Also based on the I4 hypothesis, the urn U has n balls, of which n-m have the color 1 and m have the color 2.

Because of the conjunctures of the socio-economic environment through which clients pass and which they are influencing the insurance of goods, between the random variable Z and the random variables $X_1,\dots,X_n$ there is a link, although they are defined on different fields of probability.

This link is a dependency because it refers to the fact that the values of the random variable Z change the probability with which each of the random variable $X_1,\dots,X_n$ takes values.

For probabilistic modeling of this dependence we consider the system T formed with the urns U_Z and U as well as the following hypothesis.

I5. From the T system, random extractions are executed following the following rules:

i) an extraction from the T system consists of a extraction of a ball from the urn U_Z followed by a extraction of a ball from the urn U;

ii) if the ball extracted from the urn U_z is yellow, then the urn U retains its structure before extraction a ball;

iii) (Penality) if the ball extracted from the urn U_Z is red, then in the urn U are inserted k, $k\in\mathbb{N}^*$, balls of color 1 and after that extract a ball.

Determining the distribution of the random variable *Y*

The authors want to find the distribution of the random variable Y after N extractions from the T system, using the hypothesis I5 and where $Y=\sum_{i=1}^{n} X_i$.

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First, to simplify writing, we consider $\omega_1 \equiv 0$ şi $\omega_2 \equiv 1$. As a result, B={0,1}, where the atom {0} is the analog in the binomial model (B,P(B), **P**) of the event " is randomly extracted from the urn U a ball of color 1", and the atom {1} is the analog of the event " is randomly extracted from the urn U a ball of color 2".
At the first extraction from the T system, the probabilistic model of the urn U_Z is $(B_Z, P(B_Z), \mathbf{P_Z})$, and the probabilistic model of the urn U is (B,P(B),$\{\mathbf{P}_{(b')} | b' \in B_Z\}$), where:

$$\mathbf{P}_{(b')}(\{0\})=\begin{cases}\frac{n+k-m}{n+k}, b'=0\\ \frac{n-m}{n}, b'=1\end{cases} \text{ şi } \mathbf{P}_{(b')}(\{1\})=\begin{cases}\frac{m}{n+k}, b'=0\\ \frac{m}{n}, b'=1\end{cases}.$$

Let Q_1 be the transition function of probability from $(B_Z, P(B_Z))$ to (B,P(B)). Then:
$Q_1(b', A): B_Z \times P(B) \to [0,1]$, $Q_1(b', \cdot) = \mathbf{P}_{(b')}(\cdot)$, $Q_1(b', \{b\}) = \mathbf{P}_{(b')}(\{b\})$, $b' \in B_Z, b \in B$.

It follows that the probabilistic model of the first extraction in the T system is:
$(B_Z \times B, P(B_Z \times B), \mathbf{P}_1)$, where $\mathbf{P}_1: P(B_Z \times B) \to [0,1]$, $\mathbf{P}_1(\cdot) = \mathbf{P}_Z \otimes Q_1(\cdot)$,
$\forall(b', b) \in B_Z \times B$ it is true $\mathbf{P}_1(\{(b', b)\})=\mathbf{P}_Z(\{b'\}) \cdot Q_1(b', \{b\})=\mathbf{P}_Z(\{b'\}) \cdot \mathbf{P}_{(b')}(\{b\})=$

$$=\begin{cases}\beta \cdot \frac{n+k-m}{n+k}, b'=b=0\\ \beta \cdot \frac{m}{n+k}, b'=0 \wedge b=1\\ \gamma \cdot \frac{n-m}{n}, b'=1 \wedge b=0\\ \gamma \cdot \frac{m}{n}, b'=b=1\end{cases}.$$

At the second extraction from the T system, the probabilistic model of the urn U_Z is ($B_Z \times B_Z$,P($B_Z \times B_Z$),$\mathbf{P}_Z \otimes \mathbf{P}_Z$), i.e. the product of the probabilistic model $(B_Z, P(B_Z), \mathbf{P_Z})$ with itself, because the structure of the U_Z urn is invariant to the extractions from the system T where:
$\mathbf{P}_Z \otimes \mathbf{P}_Z: P(B_Z \times B_Z) \to [0,1]$, $\mathbf{P}_Z \otimes \mathbf{P}_Z(\{(b_1', b_2')\}) = \mathbf{P}_Z(\{b_1'\}) \cdot \mathbf{P}_Z(\{b_2'\})$,$\forall(b_1', b_2') \in B_Z \times B_Z$.

The probabilistic model of the urn U at the second extraction, knowing the results of the two extractions in the urn U_Z is (B,P(B), $\{\mathbf{P}_{(b_1',b_2')} | (b_1', b_2') \in B_Z \times B_Z\}$), where:

$$\forall(b_1', b_2') \in B_Z \times B_Z,\ \mathbf{P}_{(b_1',b_2')}: P(B) \to [0,1],\ \mathbf{P}_{(b_1',b_2')}(\{0\}) = \begin{cases}\frac{n+2k-m}{n+2k}, (b_1', b_2') = (0,0)\\ \frac{n+k-m}{n+k}, (b_1', b_2') \in \{(0,1),(1,0)\},\\ \frac{n-m}{n}, (b_1', b_2') = (1,1)\end{cases}$$

$$\mathbf{P}_{(b_1',b_2')}(\{1\}) = \begin{cases}\frac{m}{n+2k}, (b_1', b_2') = (0,0)\\ \frac{m}{n+k}, (b_1', b_2') \in \{(0,1),(1,0)\}\\ \frac{m}{n}, (b_1', b_2') = (1,1)\end{cases}.$$

The probabilistic model of the urn U in the case of two extractions, knowing the results of the extractions from the urn U_Z is

$(B\times B, P(B\times B), \{\mathbf{P}_{(b'_1)} \otimes \mathbf{P}_{(b'_1,b'_2)} | (b'_1, b'_2) \in B_Z \times B_Z\})$, where:
$\mathbf{P}_{(b'_1)} \otimes \mathbf{P}_{(b'_1,b'_2)}: P(B \times B) \rightarrow [0,1], \forall (b'_1, b'_2) \in B_Z \times B_Z$,
$\mathbf{P}_{(b'_1)} \otimes \mathbf{P}_{(b'_1,b'_2)}(\{(b_1, b_2)\}) = \mathbf{P}_{(b'_1)}(\{b_1\}) \cdot \mathbf{P}_{(b'_1,b'_2)}(\{b_2\}), \forall (b'_1, b'_2) \in B_Z \times B_Z$.

As a result, the transition function of probability from $(B_Z \times B_Z, P(B_Z \times B_Z))$ to $(B \times B, P(B \times B))$, denoted by Q_2, is:
$Q_2\left((b'_1, b'_2), A\right): (B_Z \times B_Z) \times P(B \times B) \rightarrow [0,1]$, $Q_2\left((b'_1, b'_2), \cdot\right) = \mathbf{P}_{(b'_1)} \otimes \mathbf{P}_{(b'_1,b'_2)}(\cdot)$ and $\forall (b_1, b_2) \in B \times B$, it is true
$$Q_2\left((b'_1, b'_2), \{(b_1, b_2)\}\right) = \mathbf{P}_{(b'_1)}(\{b_1\}) \cdot \mathbf{P}_{(b'_1,b'_2)}(\{b_2\}) = Q_1(b'_1, \{b_1\}) \cdot \mathbf{P}_{(b'_1,b'_2)}(\{b_2\}).$$

It follows that the probabilistic model of the second extraction in the *T* system is:
$(B_Z \times B_Z \times B \times B, P(B_Z \times B_Z \times B \times B), \mathbf{P}_2)$, $\mathbf{P}_2: P(B_Z \times B_Z \times B \times B) \rightarrow [0,1]$,
$\mathbf{P}_2(\cdot) = (\mathbf{P}_Z \otimes \mathbf{P}_Z) \otimes Q_2(\cdot)$ and $\forall (b'_1, b'_2, b_1, b_2) \in B_Z \times B_Z \times B \times B$ it is true
$$\mathbf{P}_2(\{(b'_1, b'_2, b_1, b_2)\}) = \mathbf{P}_Z(\{b'_1\}) \cdot \mathbf{P}_Z(\{b'_2\}) \cdot Q_2\left((b'_1, b'_2), \{(b_1, b_2)\}\right) =$$
$$= \mathbf{P}_Z(\{b'_2\}) \cdot \mathbf{P}_1(\{(b'_1, b_1)\}) \cdot \mathbf{P}_{(b'_1,b'_2)}(\{b_2\}).$$

PROPOSITION. The probabilistic model of *N* extractions from the system *T*, $N \in \mathbb{N}^* \setminus \{1\}$, is:
$(\prod_{i=1}^N B_Z \times \prod_{i=1}^N B, P(\prod_{i=1}^N B_Z \times \prod_{i=1}^N B), \mathbf{P}_N)$,
$\mathbf{P}_N(\cdot) = (\otimes_{i=1}^N \mathbf{P}_Z) \otimes Q_N$ and $\forall (b'_1, \dots, b'_N, b_1, \dots, b_N) \in \prod_{i=1}^N B_Z \times \prod_{i=1}^N B$ it is true
$\mathbf{P}_N(\{(b'_1, \dots, b'_N, b_1, \dots, b_N)\}) = \left(\prod_{i=1}^N \mathbf{P}_Z(\{b'_i\})\right) \cdot Q_N\left((b'_1, \dots, b'_N), \{(b_1, \dots, b_N)\}\right)$, where
$$Q_N\left((b'_1, \dots, b'_N), \{(b_1, \dots, b_N)\}\right) = Q_{N-1}\left((b'_1, \dots, b'_{N-1}), \{(b_1, \dots, b_{N-1})\}\right) \cdot \mathbf{P}_{(b'_1,\dots,b'_N)}(\{b_N\})$$
and $\mathbf{P}_{(b'_1,\dots,b'_N)}(\{0\}) = \frac{n+k\cdot r_N - m}{n+k\cdot r_N}$, $\mathbf{P}_{(b'_1,\dots,b'_N)}(\{1\}) = \frac{m}{n+k\cdot r_N}$, $r_N = card\{b'_i = 0 | i = \overline{1,N}\}$,
$\mathbf{P_Z}(\{0\}) = \beta$, $\mathbf{P_Z}(\{1\}) = \gamma$.
The demonstration is done by mathematical induction in relation to *N*.

COROLLARY. For $N \in \mathbb{N}^* \setminus \{1\}$ and any $(b'_1, \dots, b'_N, b_1, \dots, b_N) \in \prod_{i=1}^N B_Z \times \prod_{i=1}^N B$, we have the formula (*):
$\mathbf{P}_N(b'_1, \dots, b'_N, b_1, \dots, b_N) = \prod_{i=1}^N \mathbf{P}_Z(\{b'_i\}) \cdot \mathbf{P}_{(b'_1)}(\{b_1\}) \cdot \mathbf{P}_{(b'_1,b'_2)}(\{b_2\}) \cdot \dots \cdot \mathbf{P}_{(b'_1,\dots,b'_N)}(\{b_N\})$.
Because
$Y: (\prod_{i=1}^N B_Z \times \prod_{i=1}^N B, P(\prod_{i=1}^N B_Z \times \prod_{i=1}^N B), \mathbf{P}_N) \rightarrow (\mathbb{R}, \mathcal{B}_{\mathbb{R}})$
the distribution of *Y* is:
$Y{:}S \cdot \begin{pmatrix} 0 & 1 & \cdots & k & \cdots & N \\ p_0 & p_1 & \cdots & p_k & \cdots & p_N \end{pmatrix}$, $p_k = \mathbf{P}_N(Y = k)$.
For $k \in \{1, \dots, N-1\}$ we define:

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$$A_k = \left\{ \left(b_{i_1}, \dots, b_{i_k}\right) \middle| \begin{array}{c} 1 \le i_1 < \cdots < i_k \le N, b_{i_j} = 1, \forall j = \overline{1,k} \wedge b_i = 0, \forall i \notin \{i_1, \dots, i_k\}, \\ i = \overline{1,N} \end{array} \right\}$$

and it is noticed that the number of subsets A_k is $\binom{N}{k}$.

Then

$$p_k = \sum_{A_k} \left(\sum_{\substack{b'_i \in \{0,1\} \\ i=\overline{1,N}}} \mathbf{P}_N\left(\left\{\left(b'_1, \dots, b'_N, b_1, \dots, b_{i_1}, \dots, b_{i_2}, \dots, b_{i_k}, \dots, b_N\right)\right\}\right) \right).$$

Also

$p_0 = \sum_{\substack{b'_i \in\{0,1\} \\ i=\overline{1,N}}} \mathbf{P}_N(\{(b'_1, \dots, b'_N, 0, \dots, 0)\}),\ p_N = \sum_{\substack{b'_i \in\{0,1\} \\ 0<i=\overline{1,N}}} \mathbf{P}_N(b'_1, \dots b'_N, 1, \dots, 1).$

It follows that the total premium of insurance is determined for *N*=*n* and it is

Pnt=$M(Y) = S \cdot \sum_{k=0}^{N} k \cdot p_k$.

Finally, the algorithm for determining the minimal threshold of the risk reserve is applied.

Numeric example

For *n*=5, *k*=1, *m*=1 and *S*=1000 u.m. to determine, with the above method, the minimal threshold of the risk reserve, knowing that $\beta=\frac{1}{5}$, $\gamma=\frac{4}{5}$ and α=0,005.

First, the distribution of the random variable *Y* must be determined, calculating the probabilities $\mathbf{P}_5$, with formula (*), where:

$\mathbf{P}_{\left(b'_1,\dots,b'_j\right)}(\{0\}) = \frac{4+r_j}{5+r_j}$, $\mathbf{P}_{\left(b'_1,\dots,b'_j\right)}(\{1\}) = \frac{1}{5+r_j}$, $1 \le j \le 5$, $r_j = \text{card}\left\{b'_i = 0 \middle| i = \overline{1,j}\right\}$.

The following distribution of the random variable *Y* is obtained:

$$Y: S \cdot \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0{,}36869 & 0{,}40809 & 0{,}1881 & 0{,}03446 & 0{,}00045 & 0{,}00021 \end{pmatrix}.$$

Then

$\text{M(Y)} = 0{,}89052\, S$; $\text{V(Y)} = 0{,}69005\, S^2$; $z_{1-0{,}005} = 2{,}58$; σ=0,83069 *S*;

$\text{R}_{\min} = 4792{,}32$ u. m..

With the classic approach, the minimal threshold of the risk reserve is get:

$$\text{R}_{\min}^{c} = 2307{,}62 \text{ m. u..}$$

The econometric approach also takes into account the risk with policyholders face, in addition to the risk producing a sinister, what justifies the inequality $\text{R}_{\min} > \text{R}_{\min}^{c}$.

Conclusion

The econometric model establish the main correlation between variable to identify the risc of insurance. So, there are a lot of factors that influence the econometric model, each one contine a calculating probabilistic.

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