

Balanced economic growth

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Abstract: *The dynamics of macroeconomic outcomes manifests on short and long periods. Analysis of economic growth is carried out for long periods of time to eliminate the effects of cyclical expansion within short periods of time. The study of economic growth process can be performed using the production function, which enables the theoretical trajectory of the economic system. When there are constant relations between the combined factors of production, economic growth trajectory is its own balanced growth. Per capita production function describes the influence of providing technical labor in the results of long-term macroeconomic dynamics. Properties of real trajectory of economic growth are dependent on the type of long term balanced growth trajectory that tends towards the real growth trajectory.*

Keywords

Economic growth, balanced economic growth, per capita production function, balanced economic growth trajectory

JEL Classification: *C02, E01*

1. Introduction

In a brief characterization it can be said that the economic growth is the change of the upward output on a long term. Economic growth is due to changes in quantitative, qualitative and structural combined inputs (labor, capital, technology). The relation between output and production factors can be represented as production function. The production function describes analytically the trajectory of the economic system evolution.

2. Modeling the economic growth process by using the production function

Macroeconomic dynamics analysis results should be made on a sufficiently long period to delineate the short-term cyclical expansion of the business cycle of economic growth which manifests itself as a dominant trend in a period of time. (Badea, 2006)

Economic growth can be defined as an increase in the production capacity of a country, identified by the sustained growth of real national income over several years. (Hardwick, Langmead, Khan, 2002)

The production function shows the relationship between the volume of inputs and outputs involved in the production of goods and services, under a given technological level (Băcescu-Cărbunaru, Băcescu, 2008):

$$Y = A(t) \cdot F(K, L) \quad (1)$$

Factors labor and capital contributions to the output (y) are highlighted by the function $F(K, L)$, which can take different forms (function Cobb –Douglas, function CES etc.) .

The symbol $A(t)$ represents the technological factor (technical progress) which is influenced by time (t) and serves multiplicative role but does not affect the marginal productivity of capital and labor factors .

The production function (1) may have different shapes (Chiriță, Scarlat, 1998):

-if we consider that the technological factor $A(t)$ influences the work and labour factors equally, the technic progress is considered neutral or Solow type:

$$Y = A(t) \cdot F(K, L)$$

-if the technological factor influences only the capital stock, the technical progress is Hicks type:

$$Y = F(A(t) \cdot K, L)$$

-if the technological factor influences only the work factor, the technical progress is Harrod type:

$$Y = F(K, A(t) \cdot L)$$

Making a logarithm and differentiating function (1) we obtain the relation between the relative change of output and relative changes of combined production factors (Badea, 2016):

$$Y = A(t) \cdot F(K, L)$$

$$\ln Y = \ln A(t) + \ln F(K, L)$$

$$\frac{dY}{Y} = \frac{dA(t)}{A(t)} + \frac{dF(K, L)}{F(K, L)}$$

$$dF(K, L) = \frac{\partial F(K, L)}{\partial K} \cdot dK + \frac{\partial F(K, L)}{\partial L} \cdot dL$$

$$\frac{\partial F(K, L)}{\partial K} = w_{mgK}; \quad \frac{\partial F(K, L)}{\partial L} = w_{mgL}$$

$$dF(K, L) = w_{mgK} \cdot dK + w_{mgL} \cdot dL$$

$$\frac{dY}{Y} = \frac{dA(t)}{A(t)} + \frac{w_{mgK} \cdot dK + w_{mgL} \cdot dL}{F(K, L)}$$

$$\frac{dY}{Y} = \frac{dA(t)}{A(t)} + \frac{w_{mgK}}{F(K, L)} \cdot dK + \frac{w_{mgL}}{F(K, L)} \cdot dL$$

$$F(K, L) = \overline{w_K} \cdot K; \quad F(K, L) = \overline{w_L} \cdot L$$

$$\frac{dY}{Y} = \frac{dA(t)}{A(t)} + \frac{w_{mgK}}{\overline{w_K} \cdot K} \cdot dK + \frac{w_{mgL}}{\overline{w_L} \cdot L} \cdot dL$$

$$\frac{w_{mgK}}{\overline{w_K}} = E_{Y-K}; \quad \frac{w_{mgL}}{\overline{w_L}} = E_{Y-L}$$

$$\frac{dY}{Y} = \frac{dA(t)}{A(t)} + \frac{w_{mgK}}{\overline{w_K}} \cdot \frac{dK}{K} + \frac{w_{mgL}}{\overline{w_L}} \cdot \frac{dL}{L}$$

$$\frac{dY}{Y} = \frac{dA(t)}{A(t)} + E_{Y-K} \cdot \frac{dK}{K} + E_{Y-L} \cdot \frac{dL}{L} \quad (2)$$

In the equations above w_{mgK} was the marginal productivity of capital, w_{mgL} was the marginal productivity of labor, $\overline{w_K}$ was the average productivity of capital, and $\overline{w_L}$ was the the average productivity of labor.

The symbol E_{Y-K} represents the elasticity of production (output) with respect to capital stock, and with E_{Y-L} was noted the production elasticity relative to labor.

If $E_{Y-K} > E_{Y-L}$ the capital stock has a greater contribution to achieving output, and if $E_{Y-L} > E_{Y-K}$ workforce has a greater contribution to achieving output (Badea, 2016).

From equation (2) results that the change of output $\frac{dY}{Y}$ is dependent on the modification of technical progress $\frac{dA(t)}{A}$, of capital stock change $\frac{dK}{K}$ and of labour change $\frac{dL}{L}$, output elasticity with respect to capital stock E_{Y-K} , output elasticity with respect to labor force E_{Y-L} (Badea, 2016).

On the short term, because only one factor is considered fluctuating and the others are constant, with the constant change of the fluctuating factor, the output change is decreasing (Law of Diminishing Returns).

On the long term all production factors are considered to be fluctuating, changes of factors is done at the same pace $\left(\frac{dA(t)}{A(t)} = \frac{dK}{K} = \frac{dL}{L}\right)$ and scale economy manifests.

Three forms of scale economy stand out (Chiriță, Scarlat, 1998):

- If the rate of output is superior to the pace of change of factors, increasing economy takes place;
- If the rate of output change is equal to the rate of change of factors, economics of constant scale takes place;
- If the rate of output change is inferior to the rate of change of factors, decreasing economy takes place.

1. Per capita production function

An important role in the economy has the technical endowment of labor (z), whose level is determined by the relationship: $z = \frac{K}{L}$.

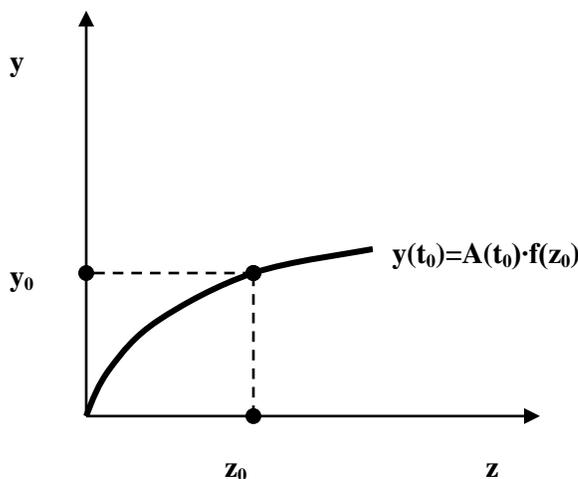


Figure 1. Graphical representation of per capita production function

How to include technical endowment of labor in the production function is a theoretical issue addressed in the works of macroeconomics theorists (Blanchard, 2003).

Using the equation $Y = A(t) \cdot F(K, L)$ we can determine output per unit of labor (*output per capita*) which we symbolize with y (Dornbusch, Fischer, 1997):

$$Y = A(t) \cdot F(K, L)$$

$$y = \frac{Y}{L} = A(t) \cdot f\left(\frac{K}{L}, 1\right) \Rightarrow y(t) = A(t) \cdot f(z)$$

Function $f(z)$ is the intensive form of aggregate production function and it represents analytically, the dependence between *output per capita* denoted by $y(t)$ and technical endowment of work related to a technology symbolized by $A(t)$:

$$f(z) = \frac{y(t)}{A(t)}$$

Because of a given moment t_0 correspond the technological level $A(t_0)$ and the technical endowment $z_0 = \frac{K_0}{L_0}$ we can write function (3) whose graphical representation is realized in fig.1.:

$$y(t_0) = A(t_0) \cdot f(z_0) \quad (3)$$

Analyzing the production function (3) stand out the influencing factors which ensure the long-term output (Badea, 2016):

- Technological change $A(t)$ for a given report of technical endowment of labour;
- Increase in the level of technical endowment of labour ($z = \frac{K}{L}$) for a given technology.

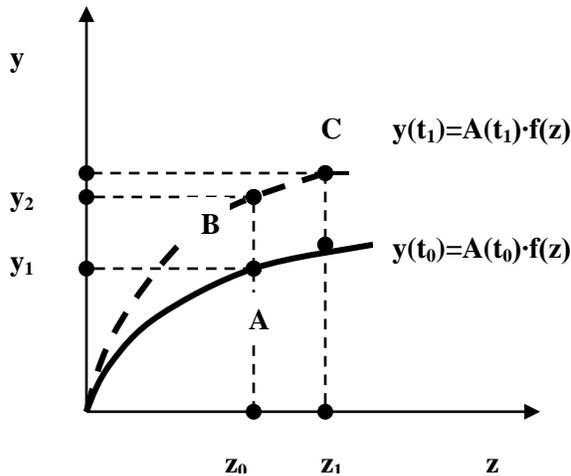


Figure 2. The influence of technological change on output

In fig. 2 technological change from $A(t_0)$ to $A(t_1)$ modifies the economy from state A to state B, the output increases from y_0 to y_1 , in the case of a constant technical endowment (z_0). If the technical endowment of labor increases from level z_0 to level z_1 the economy goes from state B to state C with the output y_2 for which there is the relationship $y_2 > y_1 > y_0$.

3. Equation of balanced economic growth

Trajectories of economic growth for which there are constant relations between the combined production factors are called trajectories of balanced growth (Chiriță, Scarlat, 1998). Next to the trajectories of balanced growth are the real trajectories of economic growth. Characteristics of real trajectory of economic growth are dependent on the type of long-term balanced growth theoretical trajectory towards which tends the real trajectory of economic growth.

Macroeconomics politics are influenced by the regularities of long-term economic growth. In 1961 Nicholas Kaldor formulates these regularities in the form of laws (Chiriță, Scarlat, 1998):

- because work measured in man-hours (L) grows slowly than the capital stock (K) and output (Y), on the long term the ratio $\frac{K}{L}$ and the ratio $\frac{Y}{L}$ increase continuously;

- the relation between the stock capital and output $\left(\frac{K}{Y}\right)$ does not have the tendency of a systematic growth, fluctuating from one period to another;
- there are regularities in remunerating the production factors and their contribution to achieving the national income (the real salary increases continuously, and the division of income between labor and capital is relatively stable).

The regularities mentioned before allow the study of economic growth trajectory for which the labor and capital factors are modified with the same rhythm (growth trajectory is considered balanced).

Using a particular shape of the production function $Y = A \cdot F(K, L)$ we can deduce the equation of balanced economic growth after doing a previous of the expression $Y = A \cdot K^\alpha \cdot L^{1-\alpha}$, where $F(K, L) = K^\alpha \cdot L^{1-\alpha}$:

$$Y = A \cdot K^\alpha \cdot L^{1-\alpha}$$

$$\ln Y = \ln(A \cdot K^\alpha \cdot L^{1-\alpha}) = \ln(A) + \ln(K^\alpha) + \ln(L^{1-\alpha})$$

$$\ln Y = \ln(A) + \alpha \cdot \ln(K) + (1-\alpha) \cdot \ln(L)$$

The final relation with logarithms differentiates to obtain a relation with infinite decimal variations¹:

$$\frac{dY}{Y} = \frac{dA}{A} + \alpha \cdot \frac{dK}{K} + (1-\alpha) \cdot \frac{dL}{L}$$

The determined equation highlights the relation between the pace of change of output and the paces of change specific to combined production factors.²

Using the differential equation obtained above we deduce an equation with finite differences according to the expression below (Dornbusch, Fischer, 1997):

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha \cdot \frac{\Delta K}{K} + (1-\alpha) \cdot \frac{\Delta L}{L} \quad (4)$$

In the equation with finite modified paces (4) sizes α and $(1-\alpha)$ have the role of *income shares* (shares for the incomes brought by the capital and labor factors at the total income) (Dornbusch, Fischer, 1997). In the equation with determined finite differences (4) there are notes with the following meanings:

¹ Infinite decimal variation=variation which tends to zero and is noted with dx; the finite variation does not tend to zero and is noted with Δx.

² The pace of change is symbolized with $\frac{dx}{x}$ or with $\frac{\Delta x}{x}$, according to the type of the variation used.

$g \rightarrow$ pace of output change (rate of economic growth);

$\frac{\Delta A}{A} = a \rightarrow$ rate of technological growth (exogenous variable);

$\alpha \cdot \frac{\Delta K}{K} \rightarrow$ accumulation of capital (size $0 \leq \alpha \leq 1$ is a subunitary elasticity coefficient of production y in relation to the capital factor);

$(1 - \alpha) \cdot \frac{\Delta L}{L} \rightarrow$ increase of labor growth (size $0 \leq 1 - \alpha \leq 1$ is a subunitary elasticity coefficient of production y in relation to the labor factor);

$\frac{\Delta L}{L} = n \rightarrow$ exogenous variable

Because we consider a balanced growth, the pace of output change equals the pace of change of capital stock:

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K} = g$$

Notations presented are replaced in the equation of balanced economic growth (4) and is determined the rate of balanced economic growth (g) [(Chiriță, Scarlat, 1998):

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha \cdot \frac{\Delta K}{K} + (1 - \alpha) \cdot \frac{\Delta L}{L}$$

$$g = a + \alpha \cdot g + (1 - \alpha) \cdot n$$

$$(1 - \alpha) \cdot g = a + (1 - \alpha) \cdot n$$

$$g = \frac{1}{(1 - \alpha)} \cdot a + n \quad (5)$$

Equation (5) describes the relation between the rate of economic growth ($g = \frac{\Delta Y}{Y}$) and the rates of production factors (technical progress $\rightarrow a = \frac{\Delta A}{A}$, labor force $\rightarrow n = \frac{\Delta L}{L}$) in terms of balanced growth ($\frac{\Delta Y}{Y} = \frac{\Delta K}{K}$).

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K}$$

Conclusion

Determining the theoretical trajectory of balanced economic growth is very important for establishing the long-term evolution of the economic system. The properties of the real trajectory of the economic growth are dependent on the type of the theoretic trajectory of long-term balanced economic growth. On the long term the real trajectory of economic growth tends towards the theoretic trajectory of balanced economic growth.

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