ANALYSIS OF ECONOMIC GROWTH DIFFERENTIAL EQUATIONS

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Abstract

The logistic growth model to explain changes in population growth rates are not. In this paper a new analysis of the population growth rate in the frequency space is described with mathematical logic and economic reasoning, so that, firstly, to a higher level of capital per capita, or at least equal to the Solow growth model reaches Second, the limits of saturation (Carrying-Capacity) is not, and ultimately, population growth rates have an impact on long-term per capita amounts. The initial classic assumption is changed in this article based on the available frequencies in the population growth equation. Finally, the last is based on the feasibility of any population growth rate with population size in the frequency space is proved.

Classification: E1, E2, E44

Key words: Solow growth model, Population growth, The Fourier series, Frequency

1. Introduction

Thomas Malthus was the first of these cycles as the population. He stated that the return pattern of demographic and economic changes in wages is due to strict law. An increased production temporarily increases wages, but wages will cause a rise in birth rates and labor force will increase again in turn, result in reduced real wages. This decrease in real wages will decrease birth rates or death rates increase. In both cases the fluctuations in population growth rate (real wage) will cause cycle.

Theory of population cycles by Simon Kuznets in the 19th century and 20th century as a model of population growth was again evaluated. So that in America, known cycles of 50 to 25 years Nasty over population growth, labor force, families, established production and inventory investment. An increase in the demand for labor may stimulate changes to technology. Return to previous growth rates may occur in the growth rate of the previous migration. The cycles themselves are produced, but also because it changes the demand for labor resulting from the creation Brown Bashed but some of the cycles themselves are produced. While the movement of population shows unimportant and irregular fluctuations which seem to be mere reflections of economic, sanitary and other conditions, we find quite clearly great waves, the main cause of which are the great wars. The deficit of births during a war and the surplus of births in the immediate postwar period repeat themselves about thirty-three years later, when the new generations are at their time of highest fertility. For the same reason thirty- three years later a third wave occurs. Of course these sub- sequent waves become broader and broader, flatter and flatter, and after a hundred years entirely interfere with each other, so that it may roughly be said that a war calls forth a set of three full cycles of a hundred years duration in all. Two things usually happen to these cycles: they are either reinforced or disturbed by cycles resulting from later wars. For the first possibility Sweden is an excellent example. There it happened again and again that at the end of a cycle a new great war renewed the waves, with the result that there was a continuous succession.

2 - The mathematical model

By introducing Fourier series, it is possible to determine the birth trajectory when the time trajectory of net reduction follows virtually any function. Let any cyclic reproductive function with period *T* be represented by the exponentiated Fourier series.

An exponentiated sinusoidal net maternity function is considered in detail, as populations with cyclically varying net maternity are of particular interest because of their connection to the Easterlin hypothesis. The dynamics of the model are largely determined by the ratio of the population's generation length *(A)* to the period of cyclist (T), and relatively simple expressions are found for the phase difference and relative amplification of the birth and net reproduction functions. More generally, an analytical expression for a population's birth trajectory is derived that applies whenever net reproductively can be written as an exponentiated Fourier series. In the cyclic model, Easterly's inverse relationship between cohort size and cohort fertility holds whenever the phase difference is zero. At other phase differences, the birth-reproduction equations have the form of predator-prey equations. The present analytical approach may thus be relevant to analyses of interacting populations(Sochen and Kim,1997).

Efforts to model populations with changing vital rates have been impeded by the lack of closed form relationships between vital rates and the resulting births. Sinusoid ally fluctuating vital rates were studied by Coale (1972) and Tuljapurkar (1985) using a Fourier series approach to the birth function. To obtain an approximate solution, however, they needed to assume a small amplitude of oscillation and consider only the first harmonic of the Fourier series.

The explanation was given about the Fourier series, it is possible that the net reproduction function is a different frequency and direction of these frequencies is a function of birth in a stable condition. As a result of the exponential Fourier series with period T and frequency ω is as follows (Sochen and Kim,1997):

$$R(t) = exp\left[\frac{1}{2}a_0 + \sum_m a_m cos(m\omega t) + \sum_m b_m sin(m\omega t)\right] \quad (1)$$

Now, the net production function and a simplified mathematical equations with respect to the relationship that exists between the birth of the path can be achieved, so that we can come to the following function:

$$g(t) = exp\left[\frac{1}{2}\left\{\frac{a_0 t}{a} + \sum_m a_m \left[\sin(m\omega t)\cot\left(\frac{1}{2}m\omega A\right) + \cos(m\omega t) - 1\right] + \sum_m b_m \left[\sin(m\omega t) + (1 - \cos(m\omega t))cott(\frac{1}{2}m\omega A)\right]\right\}\right] \quad (2)$$

Change is here in the logistic growth model and the frequency equation is applied to the growth of our workforce. It is for this we consider the three cases, the third mode is the only long-term population growth rates converge to a fixed rate to a fixed rate, others are not converging.

First case:

$$(3) L(t) = \left[\sum_m a_m cos(m\omega t) + \sum_m b_m sin(m\omega t)\right] * \boldsymbol{ae^{nt}}$$

$$\lim_{t\to\infty} n(t) = \frac{\dot{L(t)}}{L(t)} = \gamma(t)$$

This long term viewpoint on population growth rate to a fixed rate does not converge.

Second case:

$$(4) L(t) = ae^{n*}\left(\sum_m a_m cos(m\omega t) + \sum_m b_m sin(m\omega t)\right)$$

$$\lim_{t\to\infty} n(t) = \frac{\dot{L(t)}}{L(t)} = \beta(t)$$

In this case, the long-term population growth rate depends on the view point does not converge to a constant growth rate.

Third case:

$$(5) L(t) = \left[\sum_m a_m cos(m\omega t) + \sum_m b_m sin(m\omega t)\right] + \boldsymbol{ae^{nt}}$$

Frequency is limited by the number of sentences.

$$\dot{L}(t) = \frac{\partial L}{\partial t} = \sum_m c_m cos(m\omega t) + \sum_m d_m sin(m\omega t) + ane^{nt} \qquad (6)$$

$$n(t) = \frac{\dot{L}(t)}{L(t)} = \frac{\sum_m c_m cos(m\omega t) + \sum_m d_m sin(m\omega t) + ane^{nt}}{\sum_m a_m cos(m\omega t) + \sum_m b_m sin(m\omega t) + + ae^{nt}}$$

Proof that long-term in the equation is convergent to a constant growth rate is given below:

$$\lim_{t\to\infty} n(t) = \lim_{t\to\infty} F(t, \omega, m, n) =$$

$$\lim_{t\to\infty} \frac{\sum_m c_m cos(m\omega t) + \sum_m d_m \sin(m\omega t) + ane^{nt}}{\sum_m a_m cos(m\omega t) + \sum_m b_m \sin(m\omega t) + + \boldsymbol{ae^{nt}}} = \qquad (7)$$

$$\lim_{t\to\infty} \frac{\boldsymbol{ae^{nt}} \left(\frac{\sum_m c_m cos(m\omega t) + \sum_m d_m sin(m\omega t)}{ae^{nt}} + \boldsymbol{n}\right)}{\boldsymbol{ae^{nt}} \left(\frac{\sum_m a_m cos(m\omega t) + \sum_m b_m sin(m\omega t)}{ae^{nt}} + 1\right)} = n$$

We here note that the number of clauses m limit is considered. This result is very accurate is this model, the frequency rate of population growth in the long run to a growth rate of fixed convergent, but during the period of transition population growth rate oscillations is faced with the oscillations of the cosine and sine terms can be explained and this analysis is more consistent with economic realities and facts.

$$n(t) = \frac{\dot{L}(t)}{L(t)} = \frac{\sum_m c_m cos(m\omega t) + \sum_m d_m sin(m\omega t) + ane^{nt}}{\sum_m a_m cos(m\omega t) + \sum_m b_m sin(m\omega t) + + ae^{nt}} = F(t, \omega, m, n)$$

The above equation, the rate of population growth is a very interesting concept. The four parameters on population growth rate are impressive. The basic premise of classical growth only one parameter (n) has an effect on growth rate, as well as the logistic growth model only two parameters (n, t) on the growth rate affects. Such a result can gather that this is more accurate than previous assumptions and is more complete.

Another very important point in this model, it should be mentioned, is that in the long run, population growth rates will converge towards a fixed rate, that these results are compatible with the basic Solow growth model because of logistics, converge towards a long-term growth rate is zero and the model to explain changes in population growth rate assumptions of classical growth models.

Ifthepopulationgrowth rateequationto consider, namely:

$$n(t) = \frac{\dot{L}(t)}{L(t)} = \frac{\sum_m c_m cos(m\omega t) + \sum_m d_m sin(m\omega t) + ane^{nt}}{\sum_m a_m cos(m\omega t) + \sum_m b_m sin(m\omega t) + + ae^{nt}}$$

Notable in this frequency, the population growth rate is not fixed. I exchanges in population growth rate in this model can explain the frequency is better.

As is clear from the long-term population growth rates will converge to a constant. A result, all system variables (capital per capita, production per capita and per capita consumption are converging, the resulting differential equation derived from the assumption of Solow Swan convergence is important.

3 -Analysis of the Solow growth model Solow with a population frequency

The Solow growth model - Swan under the assumption that we:

$$\dot{r} = sf(r) - \big(\delta + n(t)\big)r$$

Where:

$$n(t) = F(t, \omega, m, n)$$

The main difference here, the population growth rate is not constant but a function of time is longer. In conclusion reached advanced differential equations that can not be solved simply invest the time and cannot be simply the plateau value account said.

The answer to this kind of Bernoulli equation can be expressed as a whole. In general, if the Bernoulli equation as followed:

$$\dot{x}(t) = a(t)x + b(t)$$

That solve the differential equation above is as follows

$$x(t) = e^{A(t)}\left(x_0 + \int_0^t b(\tau)e^{-A(\tau)}d\tau\right)$$

So that:

$$A(t) = \int_0^t a(\tau)d\tau$$

If the production function per capita and population growth rate in the Solow equation, we replace the frequency, we have:

$$\dot{r} + (n(t) + \delta)r = sAr^{\beta}$$

To solve the Bernoulli differential equation that we use the following variable change:

$$u = r^{1-\beta}$$

Thus we reach the following equation:

$$\dot{u}(t) = -(1-\beta)(n(t)+\delta)u(t) + (1-\beta)sA$$

The solution of this equation is as follows:

$$u(t) = e^{H(t)}\left\{ {r_0}^{1-\beta} + s(1-\beta)A\int_0^t e^{-H(t)}d\tau \right\}$$

Where H (t)is as follows:

$$H(t) = -(1-\beta)\int_0^t (n(s)+\delta)ds$$

Now, according to the population growth rate, we have:

$$n(t) = \frac{\dot{L}(t)}{L(t)} = \frac{\sum_m c_m cos(m\omega t) + \sum_m d_m sin(m\omega t) + ane^{nt}}{\sum_m a_m cos(m\omega t) + \sum_m b_m sin(m\omega t) + +ae^{nt}}$$

$$H(t) = -(1-\beta)\int_0^t \left(\frac{\sum_m c_m cos(m\omega t) + \sum_m d_m sin(m\omega t) + ane^{nt}}{\sum_m a_m cos(m\omega t) + \sum_m b_m sin(m\omega t) + +\boldsymbol{a}\boldsymbol{e}^{\boldsymbol{nt}}} + \delta \right) ds$$

$$= -(1-\beta) * \left\{ \ln\left(\sum_m a_m cos(m\omega t) + \sum_m b_m \sin(m\omega t) + +\boldsymbol{a}\boldsymbol{e}^{\boldsymbol{nt}} \right) + \delta * t \right\}$$

The answer is simple differential equation in the general case as follows:

$$u(t) = e^{H(t)}\left\{ {r_0}^{1-\beta} + s(1-\beta)A\int_0^t e^{-H(t)}d\tau \right\} =$$

$$e^{-(1-\beta)*\{\ln(\sum_m a_m cos(m\omega t) + \sum_m b_m \sin(m\omega t) + +ae^{nt}) + \delta * t\}} \left\{ {r_0}^{1-\beta} + s(1-\beta)A\int_0^t e^{(1-\beta)*\{\ln(\sum_m a_m cos(m\omega t) + \sum_m b_m \sin(m\omega t) + +ae^{nt}) + \delta * t\}} d\tau \right\}$$

$$= (\ln\left(\sum_m a_m cos(m\omega t) + \sum_m b_m \sin(m\omega t) + +\boldsymbol{a}\boldsymbol{e}^{nt}\right) + \delta * t)^{-(1-\beta)} \Big\{ {r_0}^{1-\beta} + s(1-\beta)A \int_0^t e^{(1-\beta)*\{\ln(\sum_m a_m cos(m\omega t)+\sum_m b_m \sin(m\omega t)++ae^{nt})+\delta * t\}} d\tau \Big\}$$

It can be found through numerical simulations of equation to be solved. As is clear from the equation, but its existing frequencies of Main capital per capita is certainly due to the frequency equation is population growth rate and the frequency of this type of equation is not in the capital per capita, per capita production and consumption not available. Frequency source to generate other words, the population frequencies.

We will focus on the equation of Solow - Swan in the steady state, we are applying these assumptions. The sine and cosine of these equations are bounded on the Bernoulli differential equation problem is not solved. If the analysis of the Solow growth model - Swan in the logistic growth model in the steady-state analysis of this equation, we finally reached the following equation:

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty} \frac{b(t)}{a(t)} = \lim_{t\to\infty} \frac{sA(1-\beta)}{-(1-\beta)\left(\frac{\sum_m c_m cos(m\omega t)+\sum_m d_m \sin(m\omega t)+an\,e^{nt}}{\sum_m a_m cos(m\omega t)+\sum_m b_m \sin(m\omega t)++ae^{nt}} + \delta\right)} =$$

$$\frac{sA(1-\beta)}{-(1-\beta)(n+\delta)} = -\frac{sA}{\delta+n}$$

$$\tilde{r} = \left(\frac{sA}{\delta+n}\right)^{\frac{1}{1-\beta}}$$

Conclusion

The economic growth theory usually the population growth is considered as an exponential growth rate; although this seems as a non-realistic assumption. According to Solow, the existence of a positive growth rate for population, for the purpose of explaining the economic growth is essential, but once an economic system determines its population growth path based on an exogenous rate. Any increase in the population growth rate (in ration to the previous rate) would describe lower per capita capital and production in the transition phase of the economy.

In this article we change the initial classic assumption and obtain the population growth rate based on the frequencies inherent in the population growth equation in accordance with the Four cycles. Here the Fourier series approach in adapted. To apply the Fourier series approach our equation should contain a series of frequencies, but since its derivative, that is the population growth rate has frequency therefore the main equation of population growth rate.

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